Multiple Representations: Verbal to Equations
Transcript

Grade 8 Teacher: At our last vertical team meeting, you mentioned that students were finding it challenging to write an equation from a context or to identify the correct equation for a context.

Algebra I Teacher: Yes. I'm wondering what middle grade teachers do to help students translate contexts, or verbal descriptions, into equations.

Grade 8 Teacher: You shared this problem with me *(displays problem)*.

Mr. Smith had *t* tiles. He sold 40 tiles to his neighbor. He then stacked the remaining tiles into groups of 25. What equation can be used to find *t*, the number of tiles, in terms of *s*, the number of stacks of tiles Mr. Smith made?

Algebra I Teacher: I remember.

Grade 8 Teacher: I took this problem and rewrote it for grades 5, 6, and 7 so that we can look at the diagrams they use to help students write an equation from a context. They use strip diagrams to represent contexts and then use the strip diagrams to write equations.

For this context, we know that the total number of tiles is 190. We can represent it with the long line segment.

We know that 40 tiles were given to Mr. Smith's neighbor, and those are included in the 190 tiles. We draw a rectangle underneath the 190 and section a part of it to represent these 40 tiles.

The remaining tiles will be grouped into *s* stacks. We represent this by adding a symbol that highlights the remaining part of the diagram and label it *s* for stacks of tiles.

We know that the remaining amount will be separated into groups of 25, so we mark some groups of 25 and use ellipses since we are not certain how many groups of 25 tiles there are.

In fifth grade, many students take a diagram and translate it into an equation. The diagram helps them make sure that they have accounted for each part of the problem.

Students start with the 190 and subtract the 40 to represent the tiles given to a neighbor. Then, they divide this quantity by 25 to determine the number of stacks of 25 tiles.

Algebra I Teacher: I see this in the diagram. The equation 190 – 40 leaves an amount to be stacked into groups of 25. There will be *s* groups of 25.

Grade 8 Teacher: Yes. Now, let's look at grades 6 and 7. It may seem like we are taking a step backwards given the fifth-grade problem. We are just making sure that students have a clear understanding of how to write an equation. We'll save inequalities for another time.

Here is a sixth-grade problem. Mr. Smith had 190 tiles. He sold 40 tiles to his neighbor. What equation can be used to find *t*, the number of tiles Mr. Smith still has?

Algebra I Teacher: This diagram should be simpler, because we are looking at a one-step problem.

Grade 8 Teacher: Yes, that is as far as they go in sixth grade. The diagram shows the two parts that form the whole, or the 190 tiles.

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Algebra I Teacher: I think students might write one of two equations to solve this problem. They might write 190 – 40 = *t*, or they might write 190 = 40 + *t*.

Grade 8 Teacher: We focus students on writing an equation that represents a context or situation rather than how to calculate the answer. We try to lay the foundation for more complex contexts or situations in Algebra I and beyond. In this case, we represent the context using our diagram: 190 = 40 + *t*.

Algebra I Teacher: What if a student writes an equivalent form of the equation?

Grade 8 Teacher: We return to the diagram and use it to connect the equivalent forms. This one represents the context, and this one shows how to calculate the solution. Because the grade 5 example was a two-step problem, it looks like the content addressed in grade 7.

The diagram looks the same as it did for the grade 5 problem. However, the equation looks different. In grade 5, the focus is on learning how to represent an unknown in a context with a variable. We want seventh-grade students to represent the situation rather than the solution process.

We want students to represent the two parts. The number of tiles given to Mr. Smith's neighbor, along with the amount of remaining tiles, and the whole, or the 190 tiles. The equation is 40 + 25(*s*) = 190.

Algebra I Teacher: In my original problem, Mr. Smith had *t* tiles rather than a known amount. To indicate that on the diagram, I would replace the 190 tiles with a *t* to represent *t* tiles. The same would be true of the equation. I would use *t* in place of 190, because *t* represents the total number of tiles with which Mr. Smith started.

Grade 8 Teacher: Exactly. I have one more context for us to review. In eighth grade, students represent situations using equations with variables on both sides. Mr. Smith and Mrs. Jones each have the same number tiles. Their tiles are loose or are in boxes. Mr. Smith has *b* boxes of 25 tiles and 40 loose tiles. Mrs. Jones has *b* boxes of 15 tiles and 100 loose tiles. What equation can be used to find *b*, the number of boxes that Mr. Smith and Mrs. Jones each have?

As I think about a diagram for this context, I don't know the total amount and I am not given a variable to represent it. I do know that Mr. Smith and Mrs. Jones have the same number of tiles. Because I know that, I can draw a diagram for each of them, and each diagram will have the same length.

Algebra I Teacher: I can see the loose tiles that each person has in each diagram. Mr. Smith has 40 loose tiles. Mrs. Jones has 100 loose tiles. I can also see that the top diagram represents Mr. Smith's boxes of 25 tiles. The bottom diagram represents Mrs. Jones' boxes of 15 tiles.

The expression that represents Mr. Smith's tiles is the 40 loose tiles plus *b*, the boxes of 15 tiles. Because they each have the same number of tiles, I can write these expressions as equal to each other.

Grade 8 Teacher: Yes! The diagrams help us write equations for contexts. They also help us eliminate equations when given several to choose from to represent a context or situation.

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We can look at the diagram and explain why an equation may or may not represent the context.

Algebra I Teacher: The diagram helps show that 40 is not a multiplier of *s*, so $t = 40(s) + 25$ does not make sense. It also helps show that 40 and 25 shouldn't be combined to be a multiplier of *s*.

I could see where some students might pause for a minute and want to group 40 and *s*, but the diagram clearly shows that the 25 is being repeated *s* times. The sum of 40 and *s* is not being repeated 25 times.

These diagrams are a helpful scaffold for translating a verbal context to an equation.

Grade 8 Teacher: We found an unexpected benefit when working with verbal descriptions that are completely mathematical as well. Students would read closer and understand that some terms may have more than one meaning.

Let's consider these two verbal descriptions of equations. Five more than *x* is 30 and five times more than *x* is 30. Our students often see more than and think addition. We ask them to sketch the diagrams until their proficiency increases.

Students see the relationship between the constant and the *x* when they have to draw it. The difference between an additive relationship and a multiplicative relationship becomes visible.

Algebra I Teacher: The strength of these diagrams is that they can be used vertically. Their meaning doesn't detract from the math that students will encounter later. Thank you for sharing these!

Grade 8 Teacher: You're welcome!